High Precision GNSS RTK Navigation for Soldiers and Other Military Assets

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ABSTRACT

Sub-decimetre accuracy real-time positioning and navigation for soldiers and other military assets is becoming increasingly important for a variety of missions, including relative positioning between individuals and assets. GNSS is the only system that can deliver this level of performance worldwide with limited local ground infrastructure. Methods available to achieve the above are called real-time kinematic (RTK) methods. In order for these methods to achieve sub-decimetre accuracy, some conditions are necessary, namely access to line-of-sight (LOS) signals, the absence of electronic interference, the use of phase measurements made on carrier frequencies and the use of the differential or relative mode of operation to reduce or eliminate propagation, orbital and timing errors. Carrier phase measurements are used as they have mm-level noise and cm-level multipath. Differential or relative operation involves the use of two simultaneous receivers with the aim of accurately positioning one relative to the other, as opposed to positioning them accurately with respect to a global reference frame; the two receivers can be static or moving. Differential carrier phase measurements are ambiguous and these ambiguities must be resolved either as real or integer numbers, the latter providing the higher level of accuracy due to enhanced observability. When the distance between the two receivers increases substantially, the residual effect of the atmosphere becomes a limiting factor, although dual-frequency measurements can somewhat mitigate this effect. Applications of high accuracy positioning include trajectory determination, vehicle-to-vehicle for collision avoidance and soldier-to-soldier positioning to name a few. The dominant GNSS used for the above is GPS. The use of other emerging systems such as GLONASS, Galileo and Beidou to supplement GPS in order to enhance performance is beginning to take place. The above concepts are discussed and are illustrated with examples..

1.0 INTRODUCTION

The GNSS measurements used for positioning and navigation are precisely time tagged code and carrier phase obtained through delay lock loops (DLL) and phase lock loops (PLL), respectively. Frequency lock loop (FLL) providing instantaneous Doppler estimates and signal strength measurements can also play a role in certain cases. In these notes, positioning and navigation are used interchangeably. The code measurement concept is illustrated in Figure 1; the real range ρ is needed but P, which is derived from dτ, is measured. The differences between the two are units (time versus length), satellite and receiver clock errors and, not shown in the figure, orbital and atmospheric propagation errors.

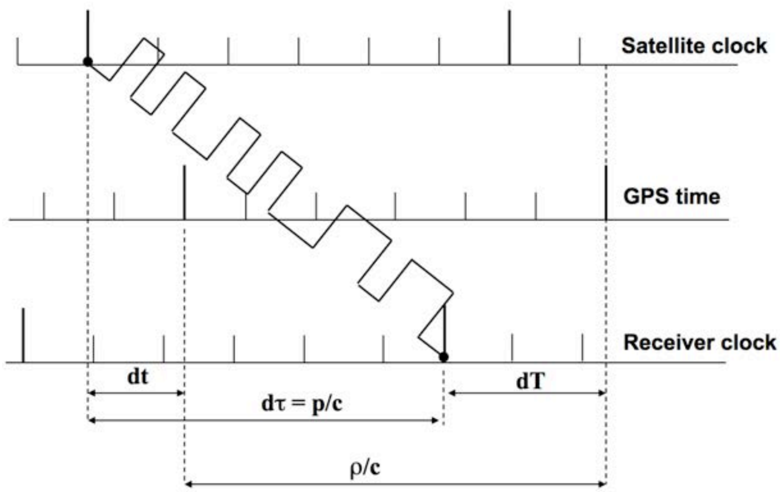

Figure 1: Code measurement concept

Code and carrier phase measurements can be algebraically expressed in units of length as

$$P = cd\tau = \rho + d\rho + c(dt - dT) + d_{ion} + d_{trop} + \varepsilon_P$$
$$\Phi = \rho + d\rho + c(dt - dT) + \lambda N - d_{ion} + d_{trop} + \varepsilon_\Phi$$

with

- P receiver code measurement
- Φ receiver carrier phase measurement
- ρ geometric range, namely $\{[x_{s1} - x_g]^2 + [y_{s1} - y_g]^2 + [z_{s1} - z_g]^2\}^{1/2}$
- subscript si refers to satellite i and subscript g to antenna (unknown)
- dρ orbital errors
- dt, dT satellite and receiver clock errors
- N cycle ambiguity (integer number)
- d_{ion} ionospheric delay
- d_{trop} tropospheric delay
- ε (C/A code noise) ≈ 5 – 200 cm for LOS (line-of-sight) measurements
- ε (P code noise) ≈ 10 cm

- ε (code multipath) $\leq$ 1 chip (non-Gaussian)
- ε_{Φ} noise {function of ε(Φ noise) and ε(Φ multipath)}
- ε(Φ noise) $\approx$ 1-5 mm (Lower than code measurements)
- ε(Φ multipath) $\leq$ 0.25 λ (Lower than code measurements

The presence of the Nλ term is due to the fact that the carrier has no time tag marking the departure from the satellite transmitter, as shown in Figure 2. At receiver lock time, the receiver PLLs measures the fractional phase and the number of cycles elapsed from that epoch onwards; however at lock time, the number of integer cycles in space between the satellite and receiver is unknown. This unknown quantity is called the carrier phase ambiguity N and is unique for each satellite-receiver pair; the wavelength λ (19 cm at L1) is used to express N in unit of length.

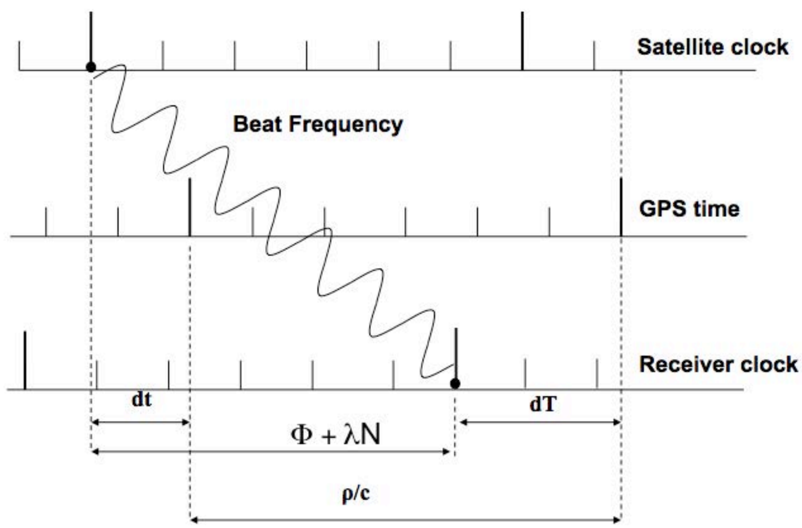

Figure 2: Carrier phase measurement concept

Low noise and low multipath make carrier phase measurements high desirable for high precision positioning, either in static or kinematic mode, despite the presence of the ambiguity term Nλ. If kinematic positioning is done in real-time, the approach is called RTK (Real-Time Kinematic) positioning. The ambiguity term, which is different for each receiver-satellite pair, can be resolved under certain conditions provided that the above errors sources can be eliminated or reduced. In order to achieve this, relative positioning is used whereby positioning of one receiver is performed with respect to another; either receiver can be in static or kinematic mode. Many differencing methods are used for different purposes and that used for RTK is usually the between-satellite-between-receiver method, also called double-difference, and illustrated in Figure 3.

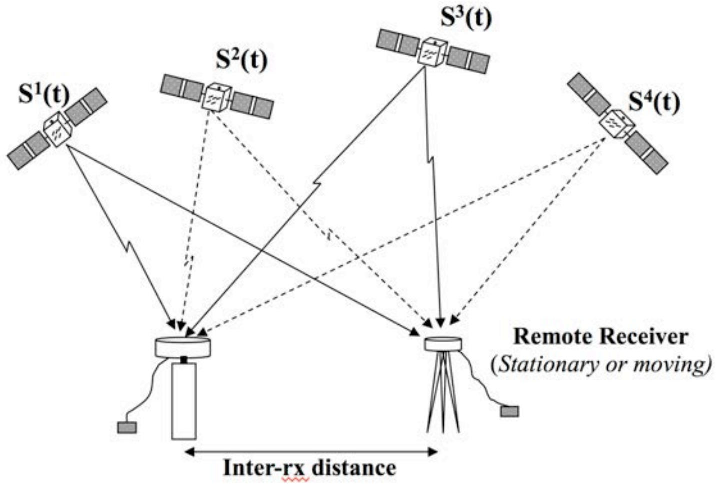

Figure 3: GNSS Double differencing

The between-satellite-between-receiver method is a linear combination of four measurements and can be written as

$$\Delta\nabla = \left\{(\bullet)_{sat_2} - (\bullet)_{sat_1}\right\}_{rx_2} - \left\{(\bullet)_{sat_2} - (\bullet)_{sat_1}\right\}_{rx_1}$$

$$\Delta\nabla P = \Delta\nabla\rho + \Delta\nabla d\rho + \Delta\nabla d_{ion} + \Delta\nabla d_{trop} + \varepsilon_{\Delta\nabla p}$$

$$\Delta\nabla\Phi = \Delta\nabla\rho + \Delta\nabla d\rho + \lambda\Delta\nabla N - \Delta\nabla d_{ion} + \Delta\nabla d_{trop} + \varepsilon_{\Delta\nabla\Phi}$$

The satellite and receiver clock errors are eliminated while the orbital and atmospheric errors are reduced, the rate of reduction being largely a function of the distance between receivers; under about 1 km, these errors are next to 0 and only the noise remains. The position differences are embedded in the term $\Delta\nabla\Phi$ contains the 3D coordinate differences. In the position solution, the coordinates of one point is held fixed by setting the partial derivatives to 0 in the design matrix and the coordinates of the second point are solved for. Disadvantages of double differencing are that (i) the variances of such linear combinations are four times those of single measurements, (ii) there is decrease in the degree of freedom, (iii) poorer geometry and (iv) the presence of serial correlation. Nevertheless the integer nature of double differenced ambiguities $\Delta\nabla N$ is preserved. If these integer ambiguities can be resolved, they can be removed from the state vector in the estimation process, resulting in better observability and accuracy. The integer values remain unchanged as long as phase lock is maintained, which is not the case when signals become weak or are temporarily unavailable, such as the case of a vehicle going through an underpass. Loss of phase lock results in a discontinuity in the cycle count by the receiver, also referred to as a cycle slip. Cycle slips are also a function of PLL algorithms and robustness and quality varies between receivers. Cycle slip detection and, if possible, recovery, is important and much related work has been carrier out on this topic; numerous methods based on phase velocity trends, L1/L2 comparisons, etc, are available and used in most software.

Measurements are not made exactly at the same time at both receivers and the difference can reach 1 ms due to clock re-synchronization delays in receivers; this issue is resolved by using the satellite coordinates for the exact (within 10-50 ns) time of measurements at each receiver, which is when the signals leave the satellites.

2.0 AMBIGUITY RESOLUTION

Ambiguity resolution using L1 only measurements is difficult (unless the distance between receivers is very short) due to the short wavelength of 19 cm; the situation is hardly better with L2 (24 cm). Hence the following linear combination of L1 and L2 measurements can be performed to increase the wavelength of derived measurements:

$$\varphi_{jk} = j\varphi_{L1} + k\varphi_{L2} \text{ with } \lambda_{jk} = c/f_{jk}$$

Most common combinations are the widelane, narrowlane and ionospheric-free, as listed in Table 1.

Table 1: Linear combinations of dual-frequency measurements

Measurement	j	k	λ(m)	Ambiguity
Widelane WL	1	-1	0.8619	$\nabla\Delta N_{WL} = \nabla\Delta N_{L1} - \nabla\Delta N_{L2}$
Narrowlane NL	1	1	0.1070	$\nabla\Delta N_{NL} = \nabla\Delta N_{L1} + \nabla\Delta N_{L2}$
Ion-free IF	1	$-f_2/f_1$	0.4844	$\nabla\Delta N_{IF} = \nabla\Delta N_{L1} - f_2/f_1 \nabla\Delta N_{L2}$
Geometry-free GF	λ_1	$-\lambda_2$	∞	$\nabla\Delta N_{GF} = \lambda_1\nabla\Delta N_{L1} - \lambda_2\nabla\Delta N_{L2}$
L1 only	1	0	0.1903	$\nabla\Delta N_{L1}$
L2 only	0	1	0.2442	$\nabla\Delta N_{L2}$

The WL combination has the longest wavelength and is therefore preferred as an initial solution. However the noise and multipath are amplified by a factor of 6.4 in this case as shown in Table 2. Hence the WL solution is used as an intermediate solution and an attempt to solve the L1 only integer ambiguities is then made. If the latter is successful, L2 ambiguities can be derived using the linear relationship

$$\nabla\Delta N_{WL} = \nabla\Delta N_{L1} - \nabla\Delta N_{L2}$$

The L1 solution can then be used if the effect of the ionosphere is found negligible. Otherwise an IF fixed solution is obtained by deriving IF ambiguities as

$$\nabla\Delta N_{IF} = \nabla\Delta N_{L1} - \frac{f_2}{f_1} \nabla\Delta N_{L2}$$

IF ambiguities are no longer integer but are still deterministic and have therefore the same advantage as integer ambiguities. IF fixed solutions are free from the ionospheric effect and are the most accurate solutions when the differential ionosphere is significant. L1, WL and IF fixed solutions for an inter-receiver distance of 109 km shown in Figure 3 (Fortes et al 1999) provide a good example of accuracy achievable in the form of double differenced residuals during a period of relatively high ionospheric activity. The above theory and sample result show that all integer ambiguity solutions are not equal and do not guaranty cm-level accuracy. Hence, when examining solutions, the type of integer ambiguities resolved should be known at the outset for a thorough evaluation, which is not always the case with commercial software.

Table 2: L1, WL and IF Errors

Error	L1 Error (m)	WL Error (m)	Ratio $\frac{WL}{L1}$	IF Error (m)	Ratio $\frac{IF}{L1}$
SV Position	$\nabla\Delta\delta\rho_{sv}$	$\nabla\Delta\delta\rho_{sv}$	1	$\nabla\Delta\delta\rho_{sv}$	1
Receiver position	$\nabla\Delta\delta\rho_{rx}$	$\nabla\Delta\delta\rho_{rec}$	1	$\nabla\Delta\delta\rho_{rec}$	1
Troposphere	$\nabla\Delta\delta\rho_{tropo}$	$\nabla\Delta\delta\rho_{tropo}$	1	$\nabla\Delta\delta\rho_{tropo}$	1
Ionosphere	$-\frac{1}{f_1^2}\nabla\Delta_{iono}$	$\frac{-\lambda_{WL}(f_1-f_2)}{cf_1f_2}\nabla\Delta_{iono} = \frac{1}{f_1f_2}\nabla\Delta_{iono}$	-1.283	0	0
Multipath	$\nabla\Delta_m$	$\frac{\lambda_{WL}}{\lambda_1}\nabla\Delta_m\sqrt{2}$	6.405	$\frac{\lambda_{IF}}{\lambda_1}\nabla\Delta_m\sqrt{\frac{f_1^2+f_2^2}{f_1^2}}$	3.227
Noise	$\nabla\Delta_\varepsilon$	$\frac{\lambda_{WL}}{\lambda_1}\nabla\Delta_\varepsilon\sqrt{2}$	6.405	$\frac{\lambda_{IF}}{\lambda_1}\nabla\Delta_\varepsilon\sqrt{\frac{f_1^2+f_2^2}{f_1^2}}$	3.227

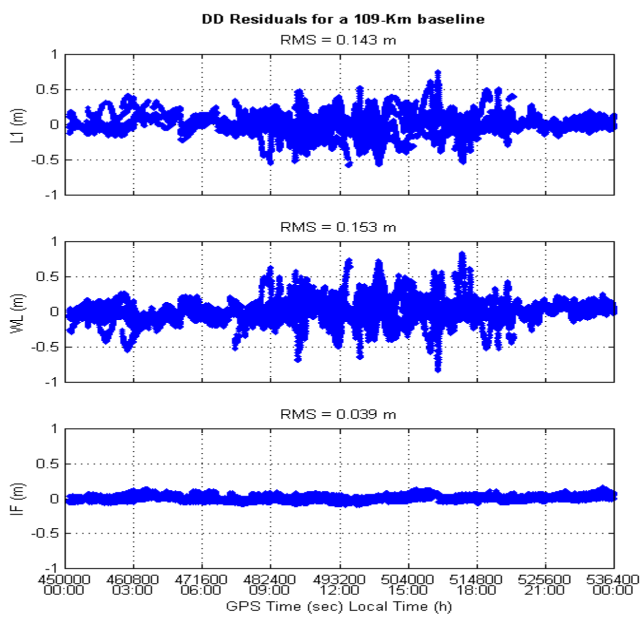

Figure 4: L1, WL and IF solutions with a separation of 109 km between receivers

In many cases, it is not possible to solve the integer ambiguities. They are then solved as real number estimates and remain part of the state vector solution as

$$x^T = \{\underbrace{\delta\phi, \delta\lambda, \delta h}_{\substack{position \\ components}}, \underbrace{\nabla\Delta N_1, \nabla\Delta N_2, \ldots, \nabla\Delta N_n}_{\substack{floating(real-valued) \\ ambiguities}}\}$$

The ambiguity values are updated each time new measurements are added and are therefore changing or “floating”; this is why solutions based on this approach are called float solutions. Convergence to the correct (but unknown ambiguities is slow as illustrated in Figure 5 and can take tens of minutes to hours, hence such solutions are not as accurate as integer ambiguity fixed solutions. The float approach can be applied to any combination of L1 and L2 measurements. If the L1 integer ambiguities cannot be resolved, it is usually due to the ionosphere in which case an IF float solution is preferred.

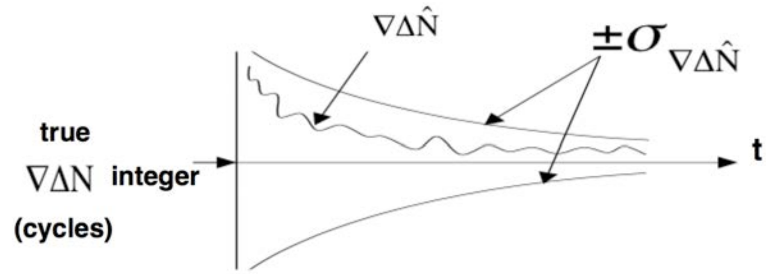

Figure 5: Convergence of float ambiguity solutions

Success in resolving integer ambiguities depend on several factors, namely

1. inter-receiver distance
2. number and geometric distribution of satellites
3. static versus kinematic
4. length of data set (to allow enough time for convergence
5. receiver DLL and PLL quality
6. frequency of cycle slips
7. availability of L2 measurements in addition to L1
8. multipath
9. differential ionospheric effect
10. multipath
11. Ambiguity resolution algorithm

The concept of integer ambiguity resolution is shown in Figure 6 for a hypothetical case of one receiver and two satellites. Possible ambiguities for each of the two satellites intersect at various grid points; the correct point has to be found for successful resolution. To improve success, the lines representing the ambiguities should intersect at a right angle to avoid a crushed grid that would result in points that are close to each other and difficult to distinguish. This happens when the multi-dimensional confidence ellipsoid is severely elongated due to high correlation between ambiguities. The LAMBDA method invented by Teunissen (1994) reduces this correlation through a re-parameterization of the ambiguity states and covariance matrix using a Z-transformation. The ambiguity states are transformed and not only minimize correlation but also reduce the search space; the latter becomes closer to a hyper-sphere, which results in a higher level of success in identifying the correct set of ambiguities. The latter is usually identified through a search based on testing a quadratic form with a Chi-square test.

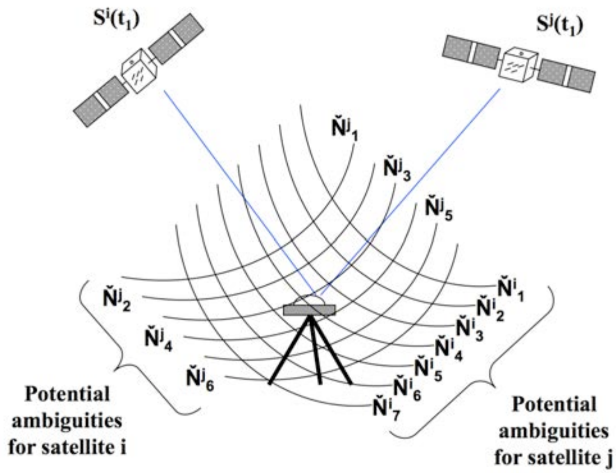

Figure 6: Ambiguity Resolution Concept

3.0 IMPLEMENTATION APPROACHES

Integer and float carrier phase ambiguity resolution methods can be implemented in various ways, depending of the application at hand. Examples are:

Static measurements taken over long periods of time: The processing of these is usually done in batch mode post-mission. Biases introduced by the ionosphere and troposphere are sometime modelled to enhance the solution accuracy. Commercial software using from 20 to 200 minutes of data provide accuracies ranging from several cm to sub-cm depending on various conditions. Advanced software packages such as Bernese (www.bernese.unibe.ch) are designed for the processing of long (multi-days to multi-months) data sequences with mm to sub-mm accuracy.

Kinematic software, post-mission and real-time: Post-mission software is usually based on forward algorithms that can also be used in real-time if needed. The post-mission version may have an option to process the data in reverse, a useful verification. Some software may also have the capability to use data from two reference receivers simultaneously for additional redundancy. Companies such as Leica, NovAtel and Trimble provide such software.

Multiple reference station approach: In order to increase RTK area coverage, a network of fixed reference stations covering an area is used to predict the atmospheric effect variations over the area. The integer ambiguities are first determined between the reference stations. The atmospheric variations are formulated in a format suitable for broadcast and incorporation into standard RTK software at the fixed or mobile receiver for which positions are needed. An example of this approach is MultiRef (Raquet 1998). A theoretically optimal but complex method to implement is that where all reference and mobile receiver data are used simultaneously (Alves 2004).

4.0 EQUIPMENT

The GNSS equipment needed for successful carrier phase positioning consists of receivers with high performance PLLs and antennas with a high phase centre stability and good multipath rejection. PLL robustness is of utmost importance to maintain phase lock under a wide range of acceleration and at least mild interference. Leading suppliers (in alphabetical order) of civilian equipment are Javad, Leica, NovAtel, Septentrio and Trimble. Prices remain relatively high, partly due to previous high non-recurring engineering costs and market characteristics. Much lower cost receivers with PLL capability are becoming available but quality so far does not match that of the above suppliers. Likewise for antennas; these supplied in conjunction with high quality receivers meet high performance requirements and are relatively large and expensive.

5.0 EXAMPLES

5.1 Precise aircraft positioning

A light jet aircraft with speeds up to 300 km/h was used by Canadian Forces, Aerospace Engineering Test Establishment, Cold Lake, Alberta, for verification of a ground-based camera tracking system in 2007. The test reported herein lasted 100 minutes over an area of 80 by 40 km. Two reference receivers separated by 40 km were placed in the above area at precisely known locations. Receivers used at these stations and on the aircraft were high quality dual-frequency civilian receivers. To independently verify the consistency and accuracy of the estimated aircraft trajectory, two independent software were used, namely NovAtel's GrafNav and the University of Calgary's FLYKIN+. The options available and used for the processing are listed in Table 3. The carrier phase ambiguities were resolved as integer values in all cases. The RMS consistency agreement in each coordinate component (North, East and Vertical) between GrafNav solutions was better than 5 cm and likewise for FLYKIN+. The second and more important comparison was between the solutions independently derived by each software; the results, summarized in Table 4, shows a RMS consistency of 2 to 8 cm in each of the three coordinates. These values also provide a good measure of the absolute accuracy of the trajectory derived by either software and option used.

Table 3: GrafNav and FLYKIN+ Software Used for Aircraft Trajectory Analysis

GPS Processing Summary (FWD = Forward Processing, REV = Reverse Processing, CMB = Forward/Reverse Combined Processing)

Software	Base Station		
	B	A	A & B
GrafNav	FWD, REV, CMB	FWD, REV, CMB	FWD, REV, CMB
FLYKIN+™ (L1&L2)	FWD	FWD	FWD
FLYKIN+™ (WL)	FWD	FWD	FWD

Table 4: GrafNav and FLYKIN+ Software Used for Aircraft Trajectory Analysis

RMS Position Agreement Between FLYKIN+™ and GrafNav Solutions

Solution 1	Solution 2	RMS Agreement (m)		
		North	East	Vertical
FLYKIN+™ L1&L2 (B)	GrafNav CMB (B)	0.033	0.025	0.075
FLYKIN+™ WL (B)	GrafNav CMB (B)	0.027	0.031	0.083
FLYKIN+™ L1&L2 (A)	GrafNav CMB (A)	0.046	0.020	0.048
FLYKIN+™ WL (A)	GrafNav CMB (A)	0.026	0.026	0.078

5.2 Relative Positioning of Moving Platforms

This case is illustrated in Figure 7 for military vehicles. It applies equally to other platforms, including soldiers. The question is whether the redundancy provided by the multiplicity of platforms can improve the speed and reliability of ambiguity resolution, under the assumption that an integer ambiguity resolution technique is applied to each pair of receiver. The answer is yes because the sum of double-differenced integer ambiguities in a close polygon (three to n sides) is zero and this knowledge can be used as a constraint. The simplest polygon to verify is a triangle encompassing three platforms. This approach, which could reasonably be implemented in RTK mode, was analysed by Luo & Lachapelle (2003) using the following measures:

- TTAF (Time to True Ambiguity Fixed)
- TAAF (Time to All Ambiguities (true) Fixed)
- Maximum time in a multi-platform configuration for a baseline to fix ambiguity
- TSR (Time Saving Rate)
- MTDW (Meantime to Detect Wrong fixes)

Improvement of up to 50% was found in the cases tested. A variation of this method is relative aircraft positioning using four receivers, with two mounted on each aircraft (Lachapelle et al 1993)

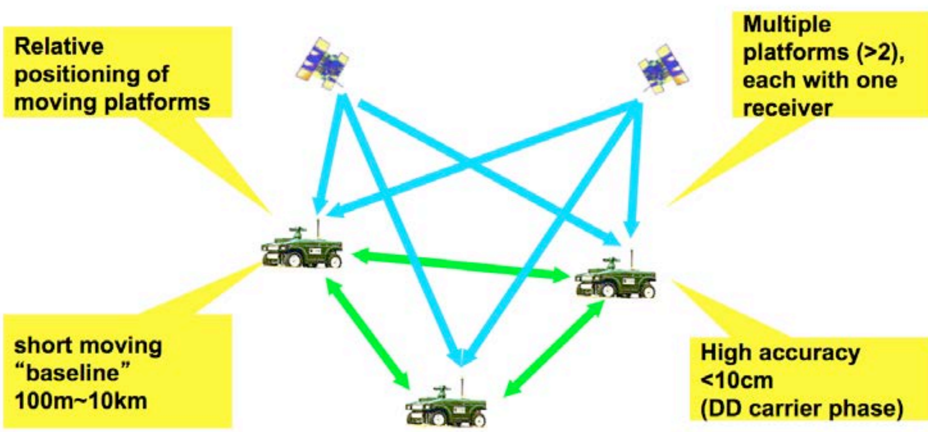

Figure 7: Moving Platform Scenario

5.3 Use of Multi-GNSS Constellation

The use of additional satellites to more effectively resolve integer ambiguities has intuitive advantages. Many PPL receiver types already have up to 150 channels to simultaneously track GPS, GLONASS, Galileo and Beidou on multiple frequencies. This allows successful ambiguity resolution is more difficult environments where signal masking angles may be high. An example of this is the precise trajectory estimation of skiers where signal masking due to the topography can easily reach 30˚ (Lachapelle et al 2009). The percentages of correctly fixed integer ambiguities for GPS, GLONASS and combined GPS-GLONASS are shown in Table 5, demonstrating clearly the advantages of additional satellites. The situation will only get better as additional constellations become available. In the example reported herein, compact NovAtel L1 GPS-GLONASS cards (less than 300 gr) with high quality PLLs were used. L1 measurements were sufficient as the fixed reference station used for differential operation was within 1 km from the mobile user, travelling at speeds up to 80 km/h in this case. Antenna selection and location for people navigation are always a challenge. In this case, Antcom antennas (67 mm diameter, 113 gr) were mounted on the user's helmet; their phase centre stability is high but the impossibility of using a ground plane in such an environment resulted in slower signal acquisition; however the PLLs produced stable carrier phase measurements after signal lock.

Table 5: GPS, GLONASS and GPS/GLONASS ambiguity resolution success on terrain with mask angle of up to 30˚

Super-imposed all around Elevation mask	% of runs with correctly fixed integer ambiguities			Comments
	GPS-only	GLONASS-only	GPS/GLONASS	
10°	57%	43%	100%	
20°	57%	43%	100%	Longer convergence times for GPS-only and GLONASS-only
30°	29%	Could not compute solutions	86%	GPS PDOP > 4 for five of seven runs

5.4 Heading Determination

Carrier phase measurements are also used to provide absolute heading and pitch using a minimum of two antennas mounted on a rigid bar fixed to a mobile platform. The 3D coordinate differences are obtained with a fixed integer ambiguity solution and rotated appropriately to obtain heading and pitch. The addition of a third antenna in a non co-linear configuration allows one to obtain roll as well. High quality, cycle slip free L1 measurements are sufficient, given the short distances between antennas. Accuracy is a function of antenna quality and inter-antenna distances. Sample heading and pitch accuracies for a two-antenna configuration as a function of the distance between the two antennas are shown in Figure 8. Numerous military applications can be made of this capability including orientation of artillery pieces. Heading can also be obtained using a single receiver and antenna for the case of a mobile user traveling in a straight line as carrier phase time differencing also provides accurate 3D position differences. The time interval selection is a function of the mobile speed (needed for sufficient virtual antenna separation); up to a few seconds is sufficient.

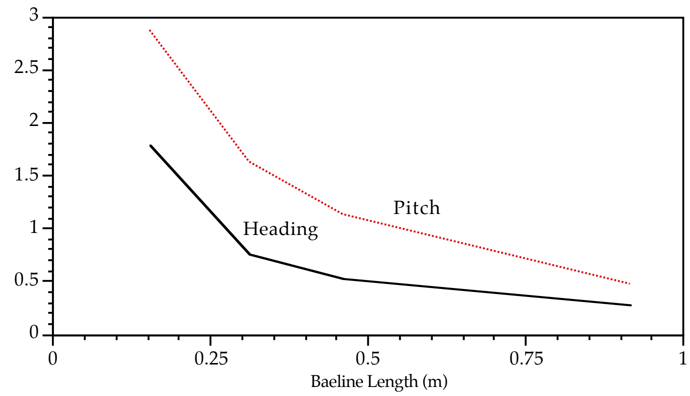

Figure 8: GNSS heading accuracy (in degrees on the vertical axis) as a function of separation (baseline) between two antennas

5.5 Ultra-Tight Integration to Enhance Carrier Phase Under Interference

Phase lock loops being less robust than DLLs, they are the first to malfunction under interference, be it an underpass, foliage, temporary obstructions or electronic interference. To complicate matters, phase re-acquisition results in new ambiguities. This is why integration of GNSS with inertial measuring units is doubly important for carrier phase positioning. Given the importance of maintaining phase lock as opposed to simply re-acquiring phase, ultra-tight (also called deep) integration is preferred as it allows aiding of PLLs by IMU measurements as opposed to simply combining unaided GNSS PLL measurements with IMU's. The concept is illustrated in Figure 9; IMU measurements contribute to the navigation solution as with other types of integration; in the ultra-tight case however, the navigation solution contributes to the local signal generator and processing of each GNSS channel, enhancing robustness in the process. Performance is a function of algorithms and their implementation, in addition to IMU measurement quality and receiver oscillator short-term stability, especially if signal integration over periods of a few hundred milliseconds is enabled to deal with weak signal environments (e.g. O'Driscoll et al 2008).

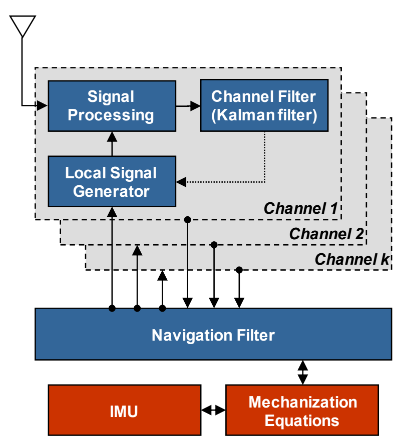

Figure 9: GNSS-IMU Ultra-tight integration concept

6.0 CONCLUSIONS

Carrier phase positioning has long proven its capability to deliver sub-decimetre performance under unobstructed visibility to satellites with the use of high performance receivers and antennas; these have traditionally been bulky and unsuitable for people use. Continuous improvements in procedures, error modelling, algorithms and software have further enhanced availability and robustness. During the past few years, lighter weight receivers and antennas with characteristics suitable for small platforms have become available, although much improvement is still needed. Given the rapid development with miniature high performance IMUs, ultra-tight integration of GNSS and inertial sensors will become more widespread and contributes to enhancing phase lock loops under interference situations. The emergence of additional satellite navigation systems, providing access to 30+ satellites at any location, will also be an important benefit. Navigation satellites in space in 2020 are shown in Figure 8. Ubiquitous carrier phase positioning is gradually becoming a reality.

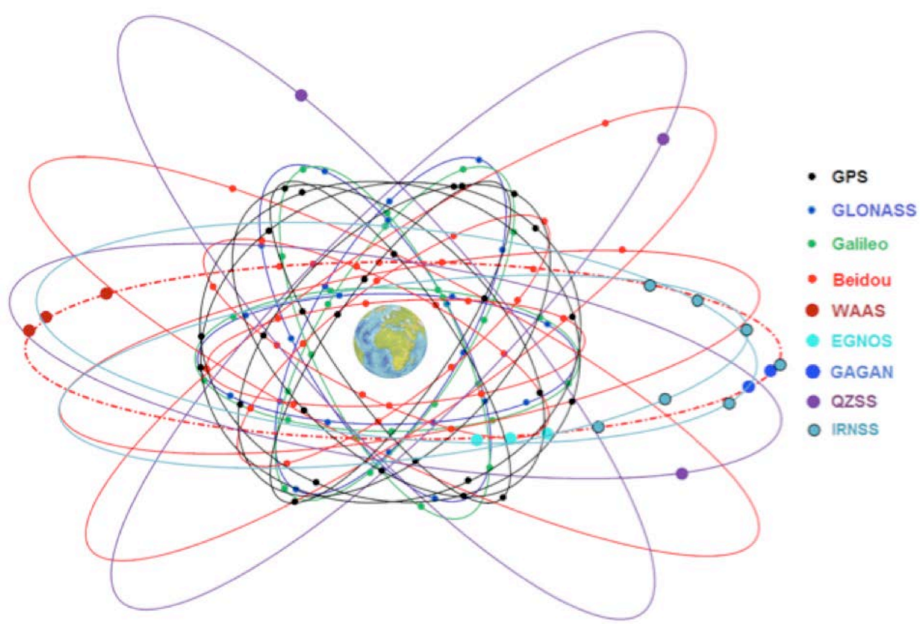

Figure 10: Navigation satellite systems in space in 2020

7.0 REFERENCES

Note: These references are available on the Internet. Many are available on PLAN.geomatics.ucalgary.ca

Alves, P. (2004) Development of two novel carrier phase-based methods for multiple Reference station positioning. PhD Thesis, published as UCGE Report No. 20203, Department of Geomatics Engineering, University of Calgary, Canada.

Fortes, L. P., G. Lachapelle, M.E. Cannon, G. Marceau, S. Ryan, S. Wee and J. Raquet (1999) Testing of a Multi-Reference GPS Station Network for Precise 3D Positioning in the St. Lawrence Seaway. Proceedings of ION GPS99, Nashville (September 13-17), pp, 1133-42.

Lachapelle G., C. Liu, and G. Lu (1993) Quadruple Single Frequency Receiver System for Ambiguity Resolution on the Fly. Proceedings of ION GPS93, The Institute of Navigation, Alexandria, VA, pp. 1167-72.

Lachapelle, G., A. Morrison, and R. Ong (2009) Ultra-Precise Sport Positioning for Sport Applications. Proceedings of 13th International Association of Institutes of Navigation, (Stockholm, 27-30Oct09), 11 pages.

Luo, N., and G. Lachapelle (2003) Relative Positioning of Multiple Moving Platforms using GPS. IEEE Transactions on Aerospace and Electronic Systems, 39, 3, 936-948.

O'Driscoll, C., M.G. Petovello and G. Lachapelle (2008), Impact of Extended Coherent Integration Times on Weak Signal RTK in an Ultra-Tight Receiver. Proceedings of NAV08 Conference, Royal Institute of Navigation, London, 28-30 October, 11 pages.

Raquet, J.F. (1998) Development of a Method for Kinematic GPS Carrier-Phase Ambiguity Resolution Using Multiple Reference Receiver, PhD Thesis, published as UCGE Report No. 20116, Department of Geomatics Engineering, University of Calgary, Canada.

Teunissen, P.J.G. (1994), *A New Method for Fast Carrier Phase Ambiguity Estimation*, Proc. of IEEE PLANS, Las Vegas, pp. 562-573.